tional couplers on anisotropic substrates," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1267–1270, 1982.
[13] T. G. Bryant and J. A. Weiss, "Parameters of microstrip transmission lines and of coupled pair of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 1021–1027, Dec. 1968.
[14] *COMPACT*, Comsat General Integrated Systems, Palo Alto, CA.

Optimum Design of 3-dB Branch-Line Couplers Using Microstrip Lines

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***Abstract* — A computer-aided design is described that makes it possible to reduce the internal impedance levels of branch-line couplers so that they may be physically constructed by microstrip lines, where the Fletcher-Powell search method has been used to optimize the design. Because microstrip lines are severely restricted in their usable impedance range, the 3-dB couplers presented here should be useful for numerous balanced-type components such as balanced mixers. The validity of the design has been experimentally verified in the microwave and millimeter-wave region.**

I. Introduction

The microstrip line is a very important transmission medium for microwave integrated circuits (MIC's) due to its reproducibility, small physical volume, light weight, and low cost. Recently, it has also been considered as a transmission medium for millimeter-wave integrated circuits [1]–[3]. However, its realizable characteristic impedance range is severely restricted, e.g., 40 Ω–140 Ω on a 0.2-mm-thick alumina substrate in *U*-band. This limited impedance range, in turn, restricts the designs of components for MIC's and millimeter-wave IC's using microstrip lines.

The directional coupler is one of the fundamental components for MIC's and millimeter-wave IC's. Especially the equal power-split (3-dB) coupler is used for balanced-type components such as balanced mixers. Among the planar structures suitable for microstrip realization, the parallel-coupled line coupler, the rat-race hybrid, and the branch-line coupler are well-known directional couplers. The parallel-coupled line coupler, however, is difficult to build for tight coupling because of the narrow gap between the microstrips. The rat-race hybrid (180° hybrid) is not so suitable for a planar structure since it has the disadvantage that the output arms are not adjacent and a crossover connection may be needed. Therefore, the branch-line coupler is most suitable for planar structures and is ideally suited for coupling values in the region of 3.0 to 6.0 dB.

The two-branch coupler, which is the most fundamental structure, has a narrow bandwidth. This disadvantage can be overcome by adding additional sections which, in theory, is an acceptable technique for broadbanding [4], [5]. In practice, this is possible for coaxial or metal waveguide structures where a wider range of impedance is possible. In microstrip, however, it is difficult to achieve more than a four-section (4-branch) coupler in Butterworth and Chebyshev designs, because the outside branch lines generally require very high impedances exceeding the upper limits of a practical realization.[1] Moreover, when the frequency becomes higher, the wide linewidths required by the low impedance lines may create an undesirable aspect ratio, due to the shortened quarter-wavelength sections. Therefore, it is difficult to realize even the two-branch and three-branch couplers in the millimeter-wave region above 50 GHz, because the center sections require very low impedances which reach the lower limits of a practical realization.

In Butterworth and Chebyshev designs, the couplers need fairly wide impedance ranges. The Zolotarev design enables the impedance ranges to be reduced to some extent [7]. A further reduction in the impedance range may be realized by applying the design method using the general form of the Chebyshev function in [8]. Although the above coupler designs can be accomplished by fully precise analytical methods, there is no assurance that the line impedances always lie within the realizable range for microstrips, because the line impedances are determined subordinately after giving the functional forms in advance.

One can solve the impedance problem by applying a computer-aided design, enabling the impedance range to be reduced effectively so that they may be physically constructed in microstrip. Furthermore, it also enables the coupling characteristics to be improved in comparison with those of the previously published couplers. The coupling characteristic was not considered positively in the previous analytical designs, because it made the design methods very complicated.

II. Realizable Impedance Range of Microstrip Line

The microstrip line has its own inherent restriction on the realizable impedance range although there is some degree of flexibility in the choice of the dielectric materials [9]–[11]. Combining the limitation of maximum substrate thickness, minimum Q factor, maximum frequency of operation, and minimum linewidth, an upper limit of impedance Z_0 of the microstrip line can be determined. The minimum Q factor, which mainly depends on conductor loss per wavelength, is proportional to the substrate thickness and square root of frequency, although the possibility of coupling to the lowest order TM surface wave limits the highest frequency of operation. With this restriction, an additional upper limit is imposed on minimum linewidth to be realized with acceptable integrity over a long length, e.g., a quarter of a wavelength. Our experience is that a minimum Q factor of 50 and minimum linewidth of 5 μm are reasonable. On the other hand, the lower impedance limit is determined by the widest linewidth to be well below a quarter-wavelength, e.g., one-eighth wavelength.

From the above limitations, the realizable ranges of impedances Z_0 as a function of substrate thickness and frequency are constrained within the range indicated by Fig. 1, where we consider the use of alumina substrates for millimeter-wave IC application [1]–[3]. The usable impedance range for alumina substrates in *C*-band (4–8 GHz) is approximately 10 Ω–100 Ω to 40 Ω–160 Ω, depending on substrate thickness. On the other hand, the usable impedance range for a 0.2-mm-thick alumina substrate[2] in *U*-band (40–60 GHz) is approximately 40 Ω–140 Ω.

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[1]A four-branch directional coupler is realized in a suspended microstrip because the suspended substrate transmission line enables one to realize high impedances up to 266 Ω [6].

[2]Minimum and maximum substrate thickness are determined by the physical strength and the possibility of coupling to the lowest order TM surface wave, respectively.

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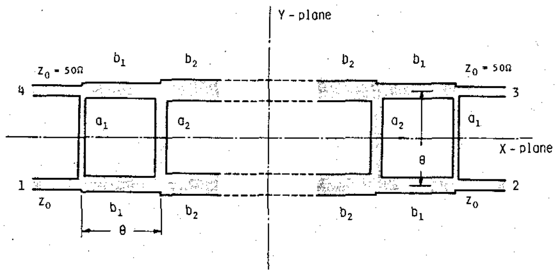

Fig. 1. A 3-dB directional branch coupler with two-fold symmetry about the x and y planes.

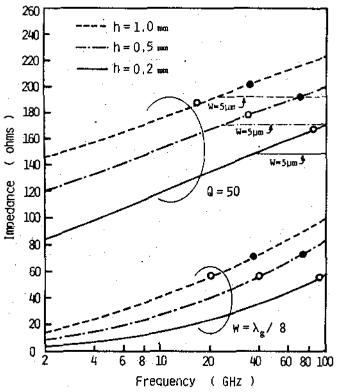

Fig. 2. Realizable impedance range of microstrip line on alumina substrates as a function of frequency. • denotes the frequency coupling to the lowest order TM surface wave, and ∘ denotes one half of the frequency of •. ($\epsilon_r = 9.8$, conductor resistivity $= 1.724 \times 10^{-5}$ Ω·mm)

The equations by which the wavelength, line impedance, and Q factor were calculated were reported by Schneider [11].

III. Method of Coupler Design

Consider a lossless reciprocal four-port branch-line coupler with two-fold symmetry about the x and y planes as shown in Fig. 2. The scattering-matrix of the coupler can be written as

$$[S] = \begin{bmatrix} S_{11} & S_{21} & S_{31} & S_{41} \\ S_{21} & S_{11} & S_{41} & S_{31} \\ S_{31} & S_{41} & S_{11} & S_{21} \\ S_{41} & S_{31} & S_{21} & S_{11} \end{bmatrix} \tag{1}$$

and the unitary relationship gives

$$|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 + |S_{41}|^2 = 1.$$

The conditions on the return loss (S_{11}), coupling (S_{21}, S_{31}), and isolation (S_{41}) for a perfect 3-dB directional coupler are

$$|S_{11}|^2 = 0 \qquad |S_{41}|^2 = 0$$
$$|S_{21}|^2 = 0.5 \qquad |S_{31}|^2 = 0.5. \tag{2}$$

However, the coupling, return loss, and isolation of a directional coupler are generally required to be within certain tolerance limits over a broad frequency band, even though the circuit may not operate perfectly at any frequencies. Although the tolerance limits for coupling, return loss, and isolation depend on the degree of performance required, e.g., less than 1.0 dB ($\approx -3.0 \pm 0.5$ dB) for the coupling imbalance, and better than -20 dB for the return loss and isolation in the case of a balanced amplifier, throughout this paper we took a tolerance limit of 0.86 dB ($\approx -3.0 \pm 0.43$ dB) for the coupling imbalance and one of -20 dB for the return loss and isolation.

By considering (2), we define a penalty function F for minimizing $|S_{11}|^1$, $|S_{41}|^2$, $|S_{21}|^2 - 0.5$, and $|S_{31}|^2 - 0.5$ as follows:

$$F(a_1, \cdots, a_n, b_1, \cdots, b_m) = \sum_{j=1}^{4} g_j$$

$$g_1 = \sum_{i=1}^{N} |S_{11}(f_i)|^2$$

$$g_2 = \sum_{i=1}^{N} \left\{ |S_{21}(f_i)|^2 - 0.5 \right\}$$

$$g_3 = \sum_{i=1}^{N} \left\{ |S_{31}(f_i)|^2 - 0.5 \right\}$$

$$g_4 = \sum_{i=1}^{N} |S_{41}(f_i)|^2 \tag{3}$$

and

$$f_i = f_0 \left(1 + \frac{i-1}{D} \right) \qquad (i = 1, \cdots, N)$$

where N is the number of sampling points, f_i's are the sampling frequencies, and f_0/D is the sampling interval. Here, all four parameters, i.e., S_{11}, S_{21}, S_{31}, and S_{41}, are used for convenience, although they are not independent for a lossless coupler. The values of the parameters of a_1 through a_n and b_1 through b_m can be obtained numerically so as to minimize the penalty function F by the Fletcher-Powell search method [12].

The optimization process was as follows.

(a) The first computation was performed without any restrictions on the line impedances by changing the sampling interval $1/D$ only.

(b) If there were some undesirably low and/or high impedance lines in the result of the first computation, the second computation was performed after one of their impedance values was changed to an appropriate fixed value.

(c) If there were still undesirably low and/or high impedance lines in the result of the second computation, the third computation was performed after two or three impedance values were held constant.

Successive computations were performed until the given tolerance limits were exceeded.

Since the degree of freedom in the circuit shape is very large in the case of the planar circuit (two-dimensional) approach [13], it is extremely difficult to determine the circuit uniquely. Thus, we took the transmission line (one-dimensional) approach here. For the couplers with large impedance steps, the effects of the junction discontinuity reactance should be added after performing a design without the junction effects. Furthermore, the electrical lengths of the circuit elements may also need to be corrected experimentally.

IV. Numerical Results

Throughout this section, BW_C, $BW_{R,I}$, k_0, $S_{\min}$, and $S_{\max}$ are defined as follows:

- BW_C the frequency bandwidth within which the coupling imbalance is better than 0.86 dB ($\approx -3.0 \pm 0.43$ dB);
- $BW_{R,I}$ the frequency bandwidth within which the return loss and isolation are better than -20 dB;
- k_0 the coupling imbalance at the center frequency;
- $S_{\min}$ the minimum return loss within the $BW_{R,I}$; and
- $S_{\max}$ the maximum ripple level of the return loss within the $BW_{R,I}$.

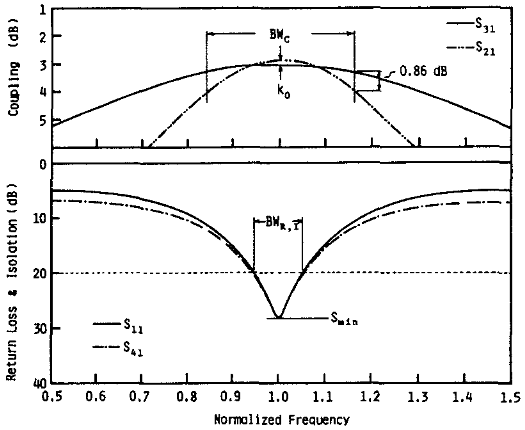

Fig. 3. Typical frequency characteristics of two-branch couplers (2-4).

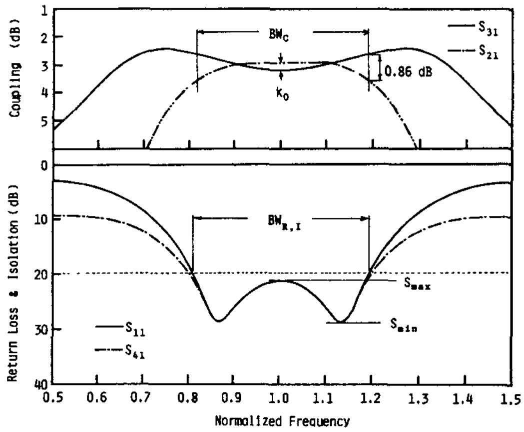

Fig. 4. Typical frequency characteristics of three-branch couplers (3-6).

TABLE I
CHARACTERISTICS OF TWO-BRANCH COUPLERS
(2-1: Conventional-Two-Branch Coupler)

	D	b_1 [Ω]	a_1 [Ω]	$BW_{R,I}$ [%]	BW_C [%]	S_{min} [dB]	k_0 [dB]
2-1	—	35.36	50.00	10	24	∞	0
2-2	20	41.18	59.50	9	41	23.24	0.44
2-3	24	39.86	57.29	10	36	25.37	0.32
2-4	30	38.55	55.13	10	32	28.26	0.21
2-5	50	36.62	52.03	10	27	36.21	0.08
2-6	100	35.67	50.50	10	25	48.51	0.02

TABLE II
CHARACTERISTICS OF THREE-BRANCH COUPLERS*
(3-1: Butterworth Coupler; 3-2: Chebysev Coupler)

	D	b_1 [Ω]	a_1 [Ω]	a_2 [Ω]	$BW_{R,I}$ [%]	BW_C [%]	S_{min} [dB]	S_{max} [dB]	k_0 [dB]
3-1	—	36.30	120.51	37.22	29	29	∞	—	0
3-2	—	34.74	99.86	41.13	39	27	50.88	20.48	0.18
3-3	18	26.95	102.88	25.26	33	39	22.73	20.86	0.18
3-4	20	26.88	105.17	24.29	32	38	23.53	22.26	0.14
3-5	20	29.42	104.95	*29.41	36	37	26.02	21.92	0.19
3-6	20	31.23	104.98	*33.33	38	37	28.89	21.80	0.23
3-7	20	34.87	105.55	*41.67	39	36	41.95	21.89	0.29
3-8	20	38.29	106.66	*50.00	39	35	35.74	22.32	0.33
3-9	18	42.35	106.72	*62 50	38	35	27.58	21.83	0.47
3-10	20	43.14	108.74	*62.50	36	34	27.49	23.34	0.39
3-11	22	43.74	110.35	*62 50	35	33	27.67	24.72	0.32
3-12	16	44.33	105.51	*71.43	38	37	25.05	20.68	0.63
3-13	18	45.46	107.97	*71.43	35	35	25.04	22.36	0.51
3-14	20	46.33	110.01	*71.43	33	33	25.46	24.00	0.41
3-15	16	47.98	106.56	*83.33	34	37	22.80	21.02	0.68
3-16	18	49.23	108.98	*83.33	31	34	23.40	22.78	0.55
3-17	20	50.15	110.84	*83.33	29	33	24.45	24.39	0.44

*The impedances of 29.41, 33.33, 41.67, 50.00, 62.50, 71.43, and 83.33 correspond to the normalized admittances of 1.7, 1.5, 1.2, 1.0, 0.8, 0.7, and 0.6, respectively.

We took four sampling points spaced f_0/D, i.e., $f_0, f_0(1+1/D)$, $f_0(1+2/D)$, and $f_0(1+3/D)$. As the frequency responses of $|S_{11}|^2$, $|S_{21}|^2$, $|S_{31}|^2$, and $|S_{41}|^2$ are symmetric about the center frequency f_0 in the lossless case, the sampling points below the center frequency f_0 are not needed. Although results have been computed for a large number of cases, the information presented in Tables I through III is restricted to the cases for which the performance of the couplers is better than that of the Butterworth coupler of the same number of branches.

A. Two-Branch Coupler

Because a two-branch coupler has too small a degree of freedom, its bandwidth cannot be improved. However, the impedance ranges required can be changed by adjusting the sampling interval $1/D$.

Fig. 3 shows the typical characteristics of a two-branch coupler (2-4 in Table I). Several couplers obtained by computer-optimization are shown in Table I, where the coupler 2-1 is a conventional two-branch coupler, and couplers 2-2 through 2-6 are obtained by changing the value of D. The characteristics of the couplers are approaching those of the conventional one as the value of D is increased (the sampling interval $1/D$ is decreased). When D is 20, the impedance range required is 41.18 Ω–59.50 Ω. On the other hand, the impedance range required by the conventional coupler is 35.36 Ω–50 Ω. The linewidths of 41.18 Ω-line and 35.36 Ω-line on a 0.2-mm thick alumina substrate are 0.276 mm and 0.362 mm, respectively, and one-eighth-wavelength at 50 GHz is about 0.283 mm. Therefore, in U-band it is extremely difficult to fabricate the conventional coupler in comparison with the coupler 2-2.

B. Three-Branch Coupler

Fig. 4 shows the typical characteristics of a three-branch coupler (3-6). The couplers obtained by the computer-optimization under several conditions are shown in Table II with the previous results by Levy [4], where the couplers 3-1 and 3-2 are the Butterworth and Chebyshev couplers, respectively. The couplers 3-5 through 3-17 were obtained after specifying the value of a_2, where the asterisk ($*$) denotes that a_2's were fixed values.

The parameters of a_1, a_2, and b_1 of couplers of 3-3 and 3-4 were optimized without any restrictions on line impedances. The impedance values of a_2 and b_1 of the couplers 3-3 and 3-4 are too low to fabricate them in microstrip. Therefore, the impedance values of a_2 and b_1 must be changed to appropriate values. The parameters a_1 and b_1 of couplers 3-5 through 3-17 were optimized after specifying the value of a_1. When the specified value of a_2 increases, the optimized value of b_1 also increases while the optimized value of a_1 is fairly constant. Therefore, one can also reduce the impedance ranges of three-branches couplers. The couplers of 3-9 through 3-17 have considerably reduced impedance ranges.

C. Four-Branch Coupler

Fig. 5 shows the typical characteristics of a four-branch coupler (4-5). The couplers obtained by the computer-optimization under several conditions are shown in Table III with the previous results by Levy [4], where the couplers 4-1 and 4-2 are the Butterworth and the Chebyshev couplers, respectively.

The parameters of a_1 through b_2 of couplers of 4-3 through 4-5

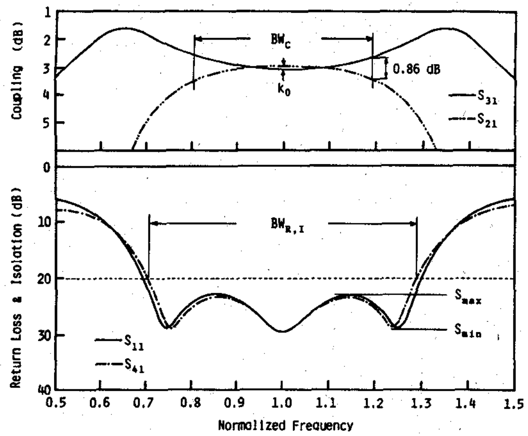

Fig. 5. Typical frequency characteristics of four-branch couplers (4-5).

TABLE III
CHARACTERISTICS OF FOUR-BRANCH COUPLERS**
(4-1: Butterworth Coupler; 4-2: Chebysev Coupler)

	D	b_1 [Ω]	b_2 [Ω]	a_1 [Ω]	a_2 [Ω]	$BW_{R,I}$ [%]	BW_C [%]	S_{min} [dB]	S_{max} [dB]	k_0 [dB]
4-1	—	39.97	30.33	263.85	52.82	43	32	∞	—	0
4-2	—	36.83	28.77	170.53	57.44	60	34	∞	22.14	0
4-3	16	44.34	39.40	152.95	90.96	63	44	36.02	20.51	0.28
4-4	20	44.14	39.15	162.39	85.32	60	40	36.23	22.73	0.19
4-5	24	45.03	41.99	157.52	91.12	58	38	29.03	23.02	0.14
4-6	16	39.40	30.66	*166.67	67.03	62	44	31.89	20.47	0.29
4-7	16	42.99	36.84	*156.25	83.91	63	44	47.30	20.45	0.28
4-8	16	47.16	44.76	*147.06	106.32	61	43	28.90	20.46	0.28
4-9	16	51.96	54.85	*138.89	136.09	57	43	23.47	20.39	0.26
4-10	16	57.37	67.39	*131.58	174.95	50	43	22.06	20.15	0.25
4-11	14	53.68	56.51	*142.86	*142.86	57	45	25.24	20.20	0.40
4-12	16	53.94	58.31	*142.86	*142.86	55	43	24.17	21.41	0.29
4-13	16	52.50	55.92	*138.89	*138.89	56	43	23.30	20.52	0.27
4-14	18	52.62	57.14	*138.89	*138.89	54	41	24.16	21.01	0.21
4-15	20	52.69	58.06	*138.89	*138.89	52	40	25.66	21.23	0.17
4-16	20	51.28	55.34	*135.14	*135.14	53	40	26.33	20.04	0.16
4-17	22	51.32	56.41	*135.14	*135.14	52	39	27.75	20.08	0.13

**The impedances of 166.67, 156.25, 147.06, 138.89, 135.14, and 131.58 correspond to the normalized admittances of 0.30, 0.32, 0.34, 0.36, 0.37, and 0.38, respectively.

were optimized without any restrictions on the line impedances. In addition to low impedances of b_1 and/or b_2, the impedance of a_1 is very high. Therefore, the impedance values of a_1 and/or a_2 must be reduced to appropriate values. The couplers 4-6 through 4-10 were optimized after specifying the value of a_1. When the specified value of a_1 decreases, the optimized value of a_2 increases, and in the case of the coupler 4-10, the impedance value of a_2 becomes higher than that of a_1. If one needs a further reduction of the impedance values of a_1 and a_2, (which also results in the reduction of the impedance range), in addition to the value of a_1, the value of a_2 must also be specified. The couplers 4-11 through 4-17 were optimized after specifying a_1 and a_2 to be equal.

V. EXPERIMENTAL RESULTS

The couplers 3-13 through 3-17 and 4-13 through 4-17 are very suitable for microstrip structures. The junction discontinuities of these couplers are not large because the impedance differences between the input/output 50-Ω lines and the main arms are small. Hence, the junction discontinuities were not considered here, and their influence did not appear clearly in the experiments. On the other hand, the ambiguity of the electrical lengths of the circuit elements is a problem one should not ignore. According to our experience, it is recognized the each one-quarter-wavelength of branch-lines is the length between the inner edges of the main arms, not between the midpoints of the main arms.

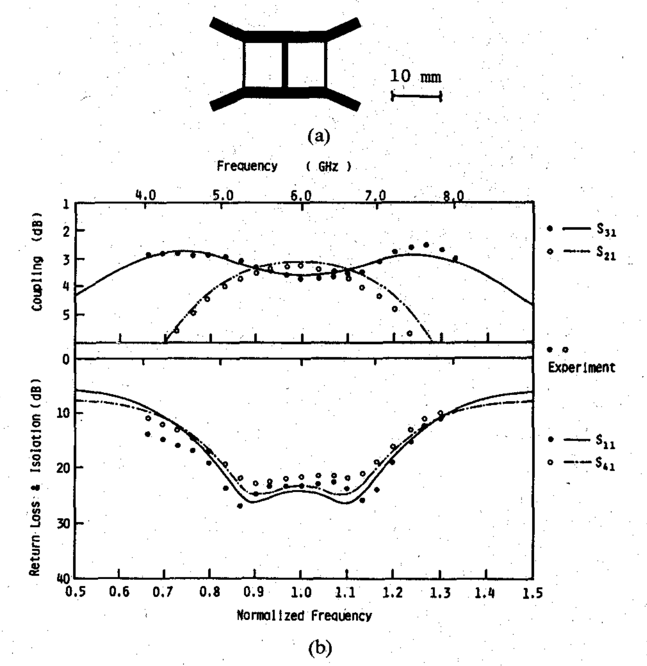

Fig. 6. (a) Circuit pattern of the coupler 3-14 on Di-Clad 522 substrate ($\epsilon_r = 2.6$) having a thickness of 0.72 mm. (b) Measured frequency characteristics of the coupler 3-14 in C-band.

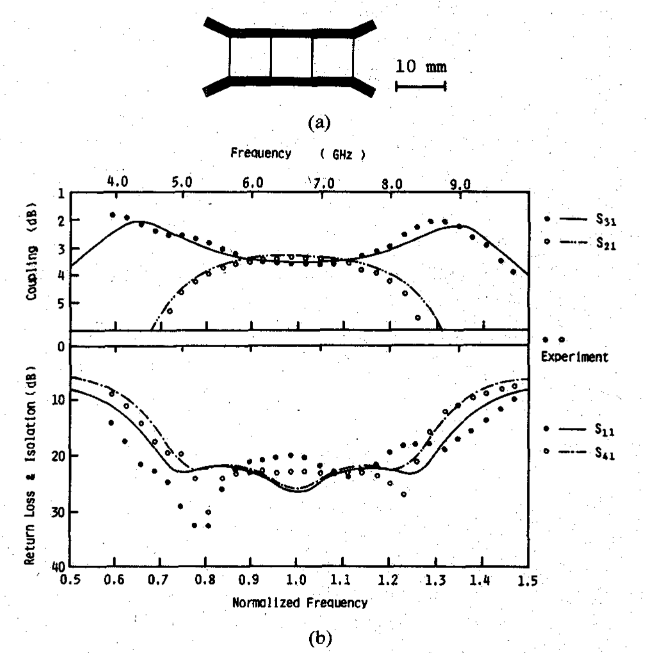

Fig. 7. (a) Circuit pattern of the coupler 4-15 on Di-Clad 522 substrate having a thickness of 0.72 mm. (b) Measured frequency characteristics of the coupler 4-15 in C-band.

Here, we fabricated three couplers, namely 3-14 and 4-15 for MIC application in C-band and the coupler 3-14 for millimeter-wave IC application in U-band. The dielectric substrates used are Di-Clad 522 ($\epsilon_r = 2.6$) having a thickness of 0.72 mm for the C-band devices, and a fine grained alumina ($\epsilon_r = 9.8$) [1] having a thickness of 0.2 mm for the U-band device.

The experimental results in the C-band are shown in Figs. 6 and 7, together with circuit patterns. The insertion loss is mainly

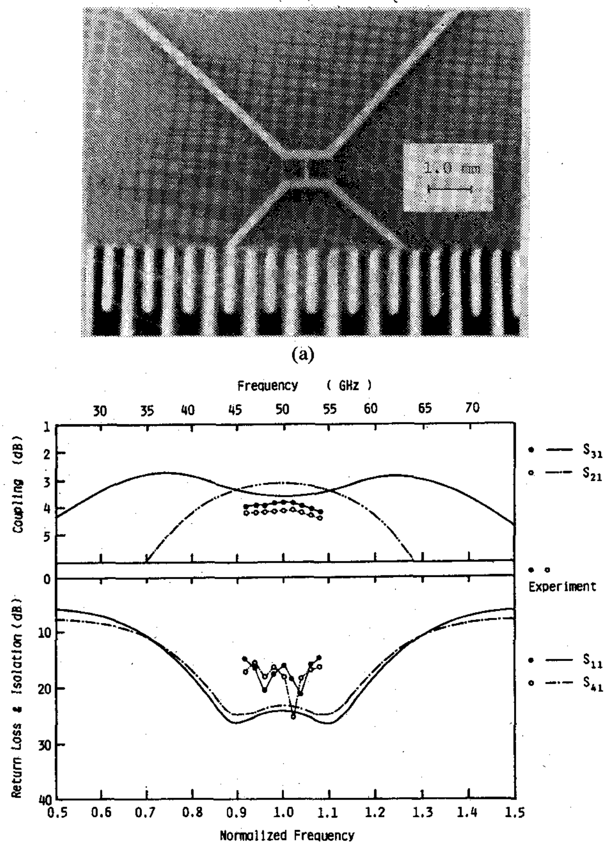

Fig. 8. (a) Photograph of the circuit pattern of the coupler 3-14 on a fine grained alumina ($\epsilon_r = 9.8$) substrate having a thickness of 0.2 mm. (b) Measured frequency characteristics of the coupler 3-14 in U-band (46–54 GHz).

due to conductor, not dielectric dissipation. Therefore, we assume that the electrical length has the following complex value:

$$\theta = \theta' - j\theta''$$

and

$$\theta''/\theta' = 0.005\sqrt{f/f_0}$$

as determined by experiments in C-band. The theoretical characteristics in Figs. 6 and 7 include the conductor loss. Therefore, the measured characteristics of the two devices in C-band showed good agreement with the theoretical ones. The frequency characteristics of the coupler 3-14 in U-band are shown in Fig. 8, together with a circuit pattern. As these characteristics include three waveguide-to-microstrip transitions and one matched load, the measured values in U-band were considerably worse than those in C-band, as shown in Fig. 6.

VI. Conclusions

Three-dB branch-line couplers with impedance ranges reduced to lie within the realizable range of microstrip line were presented. Couplers with more than four branches are not listed as they exhibit line impedances exceeding the upper limit of 160 Ω. The impedance ranges in the most practical cases of five- and six-branch couplers are 52 Ω–172 Ω and 51 Ω–208 Ω, respectively. Although only the 3-dB branch-line couplers were considered here, the design method itself is applicable to branch-line couplers with any degree and to various components in millimeter-wave IC.

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References

[1] H. Yatsuka, M. Ishizaki, T. Takano, and H. Komizo, "Millimeter-wave IC components using fine grained alumina substrate," in *1980 IEEE MTT Int. Microwave Symp. Dig.* (Washington, DC), May 1980, pp. 276–278.
[2] H. Komizo and Y. Tokumitsu, "Millimeter-wave integrated circuits," in *1981 IEEE MTT Int. Microwave Symp. Dig.* (Los Angeles, CA), June 1981, pp. 179–181.
[3] H. Ogawa, M. Akaike, M. Aikawa, T. Karaki, and J. Watanabe, "A 26-GHz band integrated circuit of a double-balanced mixer and circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 34–41, Jan. 1982.
[4] R. Levy and L. Lind, "Synthesis of symmetrical branch-guide directional couplers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 80–89, Feb. 1968.
[5] R. Levy, "Directional couplers," in *Advances in Microwaves*. New York: Academic, vol. I, 1966, pp. 115–209.
[6] A. Hislop and D. Rubin, "Suspended substrate *Ka*-band multiplexer," *Microwave J.*, pp. 73–77, June 1981.
[7] R. Levy, "Zolotarev branch-guide couplers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 95–99, Feb. 1973.
[8] R. Levy and J. Helszajn, "Specific equations for one and two section quarter-wave matching networks for stub-resistor loads," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 55–62, Jan. 1982.
[9] R. A. Pucel, "Design considerations for monolithic microwave circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 513–534, June 1981.
[10] G. D. Vendelin, "Limitations on stripline Q," *Microwave J.*, pp. 63–69, May 1970.
[11] M. V. Schneider, "Microstrip lines for microwave integrated circuits," *The Bell Syst. Tech. J.*, pp. 1421–1445, May–June 1969.
[12] M. J. D. Powell, "A method for minimizing a sum of squares of nonlinear functions without calculating derivatives," *Comput. J.*, vol. 7, pp. 303–307, 1965.
[13] T. Okoshi, T. Imai, and K. Ito, "Computer-oriented synthesis of optimum circuit pattern of 3-dB hybrid ring by the planar circuit approach," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 194–202, Mar. 1981.

Characteristic Impedance of an Oval Located Symmetrically between the Ground Planes of Finite Width

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Abstract —**A conformal transformation for the analysis of a transmission line with an oval-shaped center conductor symmetrically placed between two finite ground planes is developed. The formulation is used to calculate**

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